Unit III Static Longitudinal Stability

Aircraft Axes

The dashed lines in Figure 1 describe an aircraft's x-y-z fixed body axes, emanating from the centre of gravity. This system, with the mutually perpendicular axes in fixed reference to the aircraft, is the one most pilots recognize. The exact alignment is a bit arbitrary. Boeing sets the x-axis parallel to the floorboards in its aircraft. The geometrical plane that intersects both the x and z body axes is called the plane of symmetry, since a standard aircraft layout is symmetrical left and right (Figure 1, bottom). There are alternative axis systems (zero-lift body axis, stability axis, for example). For pilots, the wind axis system is the most useful, because it best helps in visualizing how aircraft actually behave.

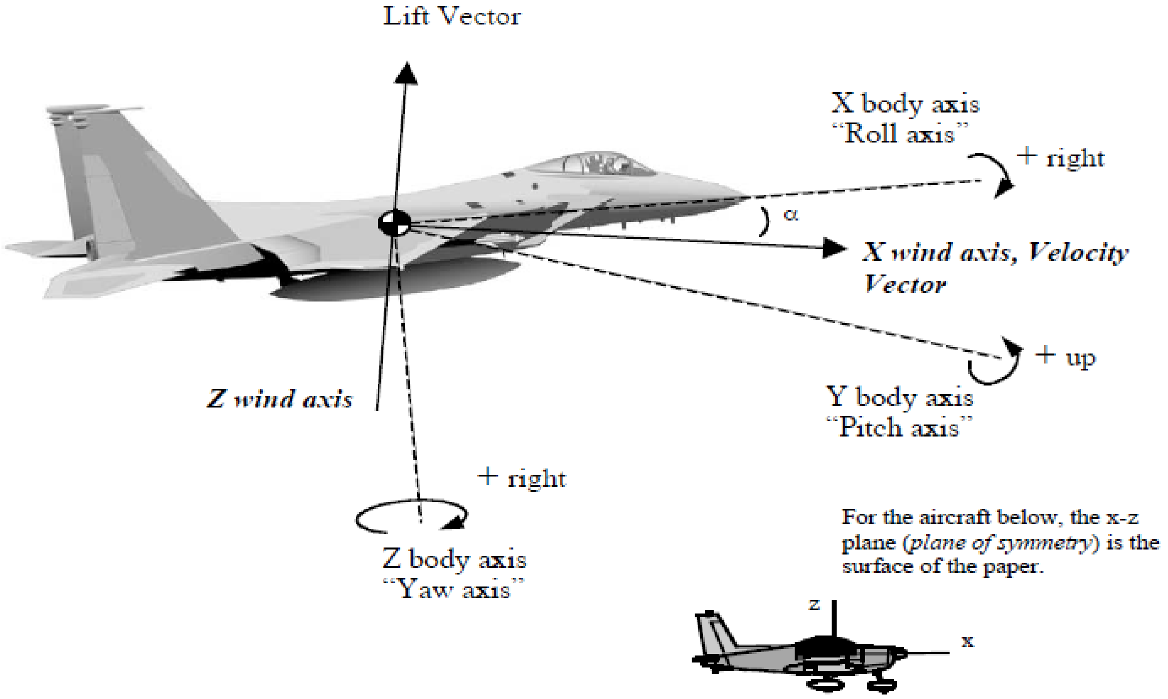


Fig. 1. Aircraft Axes

The wind axis system sets the x-axis in alignment with the aircraft's velocity vector, which points in the direction in which the aircraft is actually moving. Usually the velocity vector/wind axis lies on the aircraft's plane of symmetry, but not always. If the aircraft is in a sideslip, the velocity vector moves off the plane to some sideslip angle, β, as Figure 2 illustrates. The velocity vector also changes direction when aircraft angle of attack, α, changes.

The velocity vector is projected onto the x-z plane of symmetry for measuring α, and onto the x-y plane for measuring β. Thus it contains both α and β, as the bottom of Figure 2 shows. Both the y and z wind axes remain perpendicular to the x wind axis. So, as the velocity vector changes direction, these axes change orientation, as well. Thus they're carried along by the aircraft, but not "fixed."

Here's the essence of why the velocity vector is important to pilots: Much of aircraft response is pinned to it, both during normal flight and in unusual attitudes.

Laterally and directionally stable aircraft normally tend to roll away from, but yaw toward, the velocity vector when the vector is off the plane of symmetry. Unstable aircraft lack these instincts, or lack them in proper combination.

In addition, a trimmed, longitudinally stable aircraft tends to hold the velocity vector at a constant angle of attack, unless commanded otherwise. An unstable aircraft does not.

Aerodynamically stable aircraft tend to roll, pitch, and yaw around their respective wind axes—not around their fixed body axes, as most pilots are taught.

The picture becomes more complicated when those axes then begin to change their direction in space, but a simplified notion of wind axis rotation is often helpful in visualizing manoeuvring flight.

There's another axis system, based on the aircraft's distribution of mass: the inertia or principal axis system. The moments of inertia about the three, mutually perpendicular, principal axes determine how quickly rates of roll, pitch and yaw can change around the aircraft's centre of gravity.

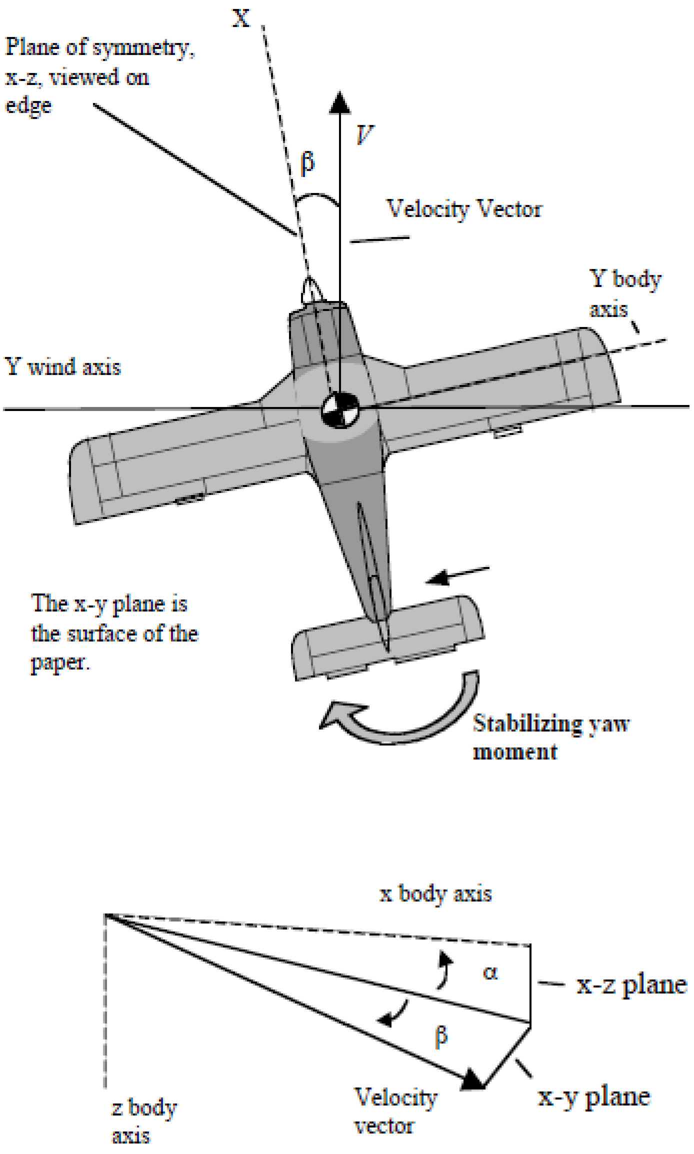


Fig. 2. Sideslip Angle, β

For example, an aircraft with tip tanks has more x-axis roll inertia when the tanks are full than when empty, and for a given airspeed, altitude, and aileron deflection will take longer to achieve a roll rate. It will also take longer to stop rolling.

The principal axes are the lines around which mass is symmetrically arranged. They may not always be shown as coincident with the aircraft fixed body axes—although, because aircraft are essentially symmetrical, they're often close enough to be considered as such. Differences in moments of inertia around each axis can lead to various coupling effects.

Lift Vector

A directionally stable aircraft returns the velocity vector to the plane of symmetry if the vector becomes displaced to some sideslip angle, β (as the "stabilizing yaw moment" is doing in Figure 2). In coordinated flight, the velocity vector lies on the plane of symmetry, as does the lift vector.

As illustrated in Figure 1, the lift vector is the upward projection of the z wind axis. Since lift is perpendicular to the air stream generated by the aircraft's velocity, it makes sense to think of its vector in wind axis terms. Fighter pilots talk cryptically of keeping the lift vector on the bogey, while an instructor might direct an inverted-attitude recovery by saying "roll the lift vector toward the sky." They generally mean a fixed vector perpendicular to the wingspan—bolted on, figuratively speaking.

That's sufficient and appropriate most of the time. The direction relative to the horizon of the lift vector so defined has a profound effect on an aircraft's maneuvering performance, but it's also possible to consider the lift vector as free to rotate around the x-axis, as it does in uncoordinated flight. For example, if a pilot uses top rudder (fuselage lift) to keep the nose up during a steep bank, the lift vector will tilt toward the high wing. Sometimes it's useful to think of the lift vector as staying oriented in space while the aircraft rotates beneath it, as it does, essentially at least, during a properly flown "slow" roll. Halfway through the slow roll, when the pilot pushes on the stick and the aircraft is producing lift inverted, the lift vector points heavenward, as it does normally. At each knife-edge, when the wings are unloaded and the pilot presses top rudder so that the fuselage is used briefly for lift, the vector still points heavenward, but out the side. We'll refer to a fixed or free lift vector, as the situation requires.

Signs, Moments, Symbols

In the sign system used with the axis notation, positive values are in the direction shown by the curved arrows in Figure 1, negative values are opposite. For example, when you pull the stick back and add left aileron, you're generating a positive pitching moment and a negative rolling moment (therefore a positive pitch rate and angle, and a negative roll rate and angle). The signs are not related to the aircraft's attitude relative to the earth or to the pull of gravity.

A moment is a force producing rotation around an axis. An aerodynamic moment is the product of a force acting on a surface—say the centre of pressure of a vertical stabilizer with a deflected rudder—times the perpendicular distance from that surface to the respective axis—the z-axis for a deflected rudder. When an aircraft is in equilibrium about an axis, all the positive and negative moments around the axis sum to zero.

The primary moments; ailerons produce rolling moments, elevators pitching moments, rudders yawing moments, but there's a further collection of direct and cross-coupled moments essential to aircraft control and often complicit in unusual attitudes.

Axis	Moment Applied	Angular Velocity	Angular Position	Moment of Inertia	Control Deflection
x	l	Roll rate, p	Roll angle φ (phi)	I_{xx} Roll Inertia	Aileron (δ_a)
y	m	Pitch rate, q	Pitch angle θ (theta)	I_{yy} Pitch Inertia	Elevator (δ_e)
z	n	Yaw rate, r	Yaw angle ψ (psi)	I_{zz} Yaw Inertia	Rudder (δ_r)

For reference, the table above shows notations used for moments, angular velocities, angular positions, moments of inertia, and control deflections about each axis.

Notice the preference for arranging things by alphabetical order. Thus the letters don't always mean what your mnemonically inclined brain would like them to mean ("r" doesn't stand for roll rate; "p" doesn't stand for pitch rate, and, while "L" stands for lift, a lowercase "l" stands for roll moment).

Stability and Control Derivatives

Moments about the axes drive aircraft attitude. Stability is the tendency of an aircraft, to generate the aerodynamic moments necessary to return it to its original equilibrium, when disturbed. During unusual attitudes, if an aircraft is left to its hands-off free response, those same moments can become destabilizing. At high bank angles, for example, directional stability causes the nose to descend below the horizon and speed to increase. When an aircraft is inverted, longitudinal stability causes the nose to fall below the horizon, as well. And at angles of attack past stall, rolling moments that would ordinarily damp out can instead produce autorotation and spin departure.

In normal maneuvering in a stable aircraft, a pilot uses the controls to overcome the aircraft's stabilizing moments and to establish a new equilibrium, at least temporarily. This may be easy or not so easy, depending on the degree of inherent stability and the availability of control power to do the job.

Stability and control are measured in terms of derivatives—the rate of change of one variable with change in another. The rates of change of moments in pitch, roll, and yaw can vary with angle of attack, sideslip angle, the presence of aerodynamic and/or inertial couples, control deflections, and with airspeed.

Aerodynamic Derivatives for Roll

Aerodynamic Stability Derivative Symbol	**Name**	**Description**
$-C_{l_\beta}$ C = coefficient l = roll moment β = sideslip angle	Rolling moment due to sideslip. (Lateral stability produced by dihedral effect)	Aircraft rolls away from the direction of sideslip. Main causes are geometrical dihedral and/or wing sweep, and fuselage-induced airflow changes that place the wings at different angles of attack. ✈ Roll due to sideslip is proportional to sideslip angle, β, and to the coefficient of lift, C_L, up to the stall, but may vary afterwards. ✈ Roll rate commanded by aileron / spoilers is affected by sideslip angle and direction. ✈ Wingtip washout, and / or flap deployment, reduce $-C_{l_\beta}$. ✈ Depends on wing position relative to fuselage. ✈ Decreased by wing taper and low aspect ratio (wingspan^2/wing area).
$-C_{l_p}$ l = roll moment p = roll rate	Rolling moment due to roll rate. (Roll damping)	As an aircraft rolls in response to a disturbance, the angle of attack increases on the down-going wing and decreases on the up-going wing. The resulting change in lift produces an opposing rolling moment. The aircraft stops rolling. If the pilot holds aileron deflection, roll damping moment builds until it's equal to the opposing moment produced by the aileron deflection. Roll rate then becomes constant. ✈ Roll damping disappears on wing sections at stall; autorotation is the reversal of roll damping. ✈ Damping increases with the slope of the C_L curve. ✈ Reduced by low aspect ratios and/or wing taper. ✈ Roll damping decreases with altitude.

$+C_{l_r}$ l = roll moment r = yaw rate	Rolling moment due to yaw rate.	Yaw rate causes airflow velocity to increase on the advancing wing and decrease on the retreating wing, causing a span wise change in lift and a rolling moment. ✈ The effect follows the lift curve, becoming greatest at C_{Lmax} and then falling off after the stall. (C_{l_r} = approx. $C_L/4$). ✈ Rolling moment due to yaw rate contributes to spiral instability and to spin departure. ✈ When entering a sideslip, rolling moments due to the temporary yaw rate and the growing sideslip angle are additive. ✈ Wingtip washout, and/or flap deployment, reduces C_{l_r}. ✈ Little affected by wing position on fuselage. ✈ Increases with aspect ratio, decreases with wing taper. ✈ Varies with the square of the difference in tip speed (since lift varies with V^2).

Aerodynamic Derivatives for Yaw

Aerodynamic Stability Derivative Symbol	**Name**	**Description**
$+C_{n_\beta}$ n = yaw moment β = sideslip angle	Yawing moment due to sideslip. (Directional stability)	Also known as weathercock stability. Aircraft yaws toward the direction of sideslip to align the longitudinal, x-axis with the relative wind. ✈ The fuselage alone is usually destabilizing; principal stability contribution comes from vertical tail, although swept wings are stabilizing, an effect that increases with C_L. ✈ Spiral instability occurs when directional stability is high and lateral stability is low. ✈ Low directional stability and high lateral stability promotes Dutch roll.

$-C_{n_p}$ n = yaw moment p = roll rate	Yawing moment due to roll rate.	The induced change in angle of attack on a rolling wing causes the lift vector to tilt back on the wing going up, and forward on the wing going down. This adds components of thrust and drag, which produce a yawing moment opposite the direction of roll (similar to adverse aileron yaw). ✈ Increases with aspect ratio, roll rate. ✈ Increases with C_L. (C_{n_p} = approx. $C_L/8$). ✈ Wingtip washout, and / or flap deployment, reduces C_{n_p}. ✈ Largely independent of taper. ✈ Reverses effect when the wing goes into autorotation.
$-C_{n_r}$ n = yaw moment r = yaw rate	Yawing moment due to yaw rate. (Yaw damping)	When an aircraft has a yaw rate, opposing aerodynamic damping forces build up ahead and behind the centre of gravity. ✈ Main contribution comes from the vertical tail, but the forward fuselage can also contribute (unlike C_{n_p}, in which the fuselage forward of the wing is destabilizing). ✈ Wings also contribute, since the advancing wing produces more induced and profile drag than the retreating wing. ✈ Wing contribution to yaw damping increases with angle of attack; the tail's contribution may decrease due to disrupted airflow at high α. ✈ Yaw damping decreases with altitude.

Rudder/Aileron Cross Derivatives

Control Derivative Symbol	Name	Description
$C_{l_{\delta_r}}$ l = roll moment δ = deflection r = rudder	Rolling moment due to rudder deflection.	A roll moment is produced if the lift generated by rudder deflection acts at a point above the roll axis. Right rudder, for example, produces a left rolling moment. This can become apparent in aircraft without dihedral effect. ✈ Diminishes as angle of attack increases.
$C_{n_{\delta_a}}$ n = yaw moment δ = deflection a = aileron	Yawing moment due to aileron deflection. (Adverse yaw)	An aileron deflected down creates more induced drag than the opposite aileron deflected up. The result is a yawing moment opposite the direction of bank. Profile drag increases on both wings when the ailerons are deflected, the difference depending on aileron design. ✈ Adverse yaw increases with wing angle of attack, because drag rises faster than lift at high α. ✈ Spoilers for roll control can produce proverse yaw. ✈ Differential ailerons or Frise ailerons counteract adverse yaw with opposing drag—although their primary function is to lower aileron control force.

Pitch Damping

Aerodynamic Derivative Symbol	**Name**	**Description**
C_{m_q} m = pitching moment q = pitch rate	Pitching moment due to pitch rate. (Pitch damping)	When aircraft pitches up or down, the motion of the horizontal stabilizer causes a change in the stabilizer's angle of attack, which generates an opposing, or damping, pitching moment. ✈ Pitch-damping moment increases with pitch rate (and thus with g load). ✈ Pitching moment due to pitch rate affects short-period response and stick force per g in pull-ups and turns. ✈ Pitch damping decreases with altitude. ✈ Pitch damping increases with increased distance between the horizontal stabilizer and the aircraft centre of gravity.

Axes and Derivatives

Longitudinal Static Stability

An aircraft has positive longitudinal static stability if its initial response in pitch, in 1-g flight, is to return to equilibrium around its trim point after displacement by a gust or by the temporary movement of the elevator control.

When you trim an aircraft to fly at a given coefficient of lift, C_L, but then push or pull on the stick and hold it there in order to fly at a different C_L you're working against the aircraft's inherent stability. The aircraft generates a restoring moment that's proportional, if you don't retrim, to the force you feel against your hand. The faster that force rises with stick deflection, the more stable your aircraft.

Classical stability depends on the distance between the aircraft's center of gravity and a set of neutral points farther aft along the longitudinal axis—the larger the distance between c.g. and neutral point the higher the stability.

An aircraft in trim is in an equilibrium state around its pitch, or y, axis. All the competing up or down moments (see Figure 6) generated by the various parts of the aircraft, and acting around its c.g., are in balance. In aerodynamics notation, a pitch-down moment carries a negative sign; pitch up is positive. In equilibrium, all moments sum to zero.

Figure 3, top, shows the change in coefficient of moment in pitch, C_M, which results from a change in coefficient of lift for a statically stable aircraft.

The longitudinal static stability curve crosses the C_L axis at the trim point, where $C_M = 0$. If the relative wind is displaced by a temporary gust or a pull on the stick, so that the C_L of the wing goes up to point A, a negative pitching moment results, B, which restores the aircraft to its trimmed angle of attack, α, and thus C_L.

The bottom of Figure 3 shows how the stability curve moves vertically when you change elevator angle to fly at a different C_L. Note that the aircraft's stability remains the same, but the trim point shifts.

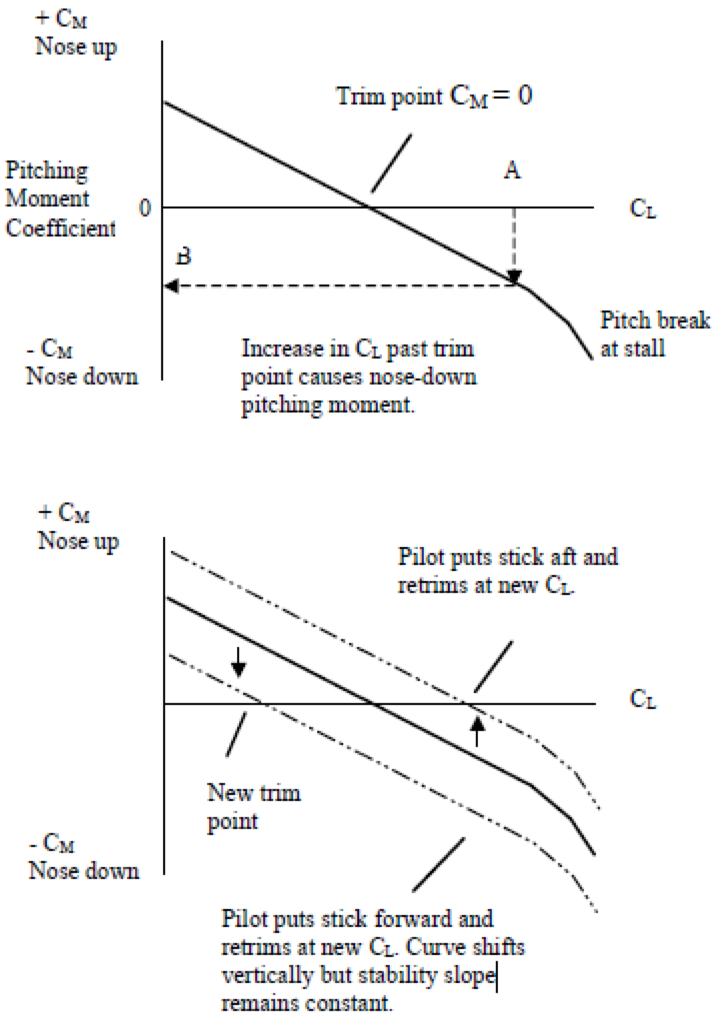


Fig. 3. Pitching Moment versus C_L

The stability curve typically takes a downward turn to a more negative slope as the aircraft passes the stalling angle of attack. This is because the downwash at the tail decreases as the wing gives up lift, and because the pitching moment of the wing itself becomes more negative as its center of pressure suddenly moves rearward at the stall. The increase in downward pitching moment, -C_M, is helpful since it aids stall recovery.

A negative slope is necessary for positive static stability. The more negative the slope the more stable the aircraft. In addition, there must be a positive pitching moment, C_M, associated with $C_L = 0$. The curve for a neutrally stable aircraft has a zero slope; so no change in pitching moment results from a change in angle of attack (Figure 4).

The stability curve for a statically unstable aircraft has a positive slope. For normal certification, it must be necessary to pull in order to obtain and hold a speed below the aircraft's trim speed, and push to obtain and hold a speed above trim speed. A statically unstable aircraft doesn't obey this (Figure 5). Instead, a change in angle of attack from trim leads to a pitching moment that takes the aircraft farther from equilibrium, and actually produces a reversal in the direction of stick forces.

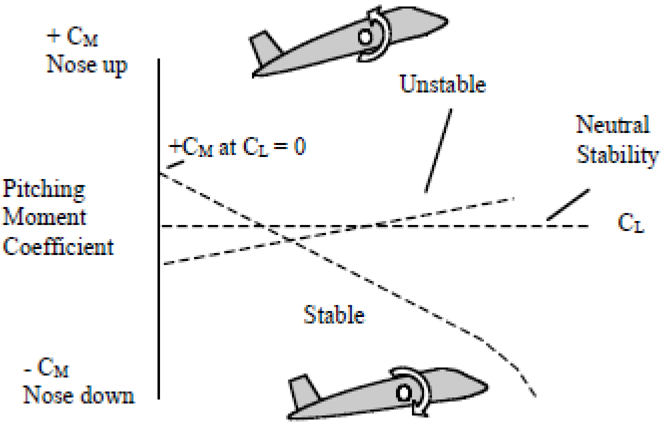


Fig. 4. Curve Slope

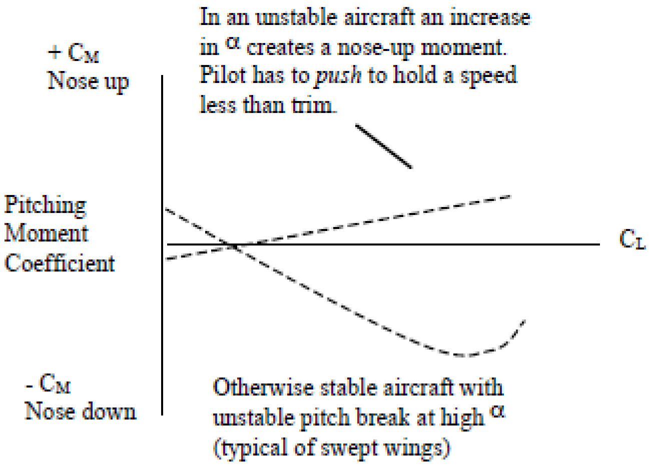


Fig. 5. Examples of Instability

The result of moderate instability might still be a flyable aircraft, but the workload goes up. Look at the positive, unstable slope in Figure 5. If you pulled back on the stick the aircraft would pitch up and slow. But if you then let go of the stick the nose would continue to pitch up, since a positive pitching moment would remain. It would require a push force to maintain your climb angle, not the mandated pull. If you pitched down and let go, the nose would tend to tuck under. You'd have to apply a pull force to hold your dive angle, not the mandated push.

Pilots experience longitudinal static stability most directly through the control force needed to change the aircraft's equilibrium from one airspeed trim point to another. The steeper the slope of the stability curve, the more force needed.

High performance, competition aerobatic aircraft tend to be somewhere on the stable side of neutral. Compared to other types, aerobatic aircraft can feel twitchy at first, partly because the light control forces associated with their shallow ΔCM/ΔCL curves cause pilots to over control. But compared to aerobatic types, more stable aircraft can feel stiff and reluctant.

Figure 6 shows how the different parts of an aircraft contribute to longitudinal stability characteristics. The fuselage and the wing are destabilizing. Static stability depends on the restoring moment supplied by the horizontal tail being greater than the destabilizing moments caused by the other parts of the aircraft. If you require an aircraft with a wide center of gravity loading range, make sure to give it a powerful enough tail (large area, large distance from c.g., both) to supply the necessary restoring moments.

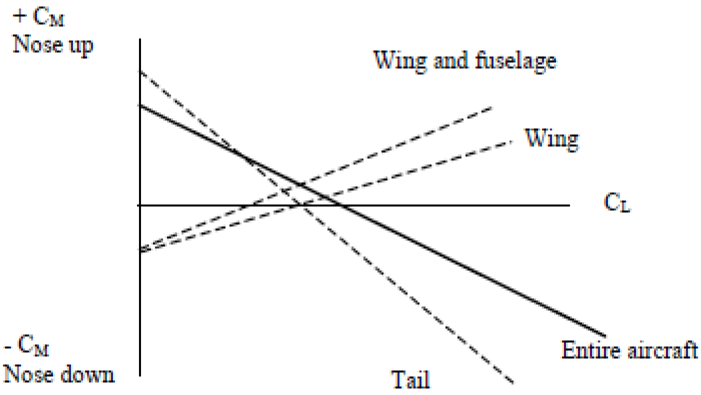


Fig. 6. Stability Components

On conventional aircraft, once the design is set, static longitudinal stability and the control force necessary to overcome that stability are both functions of aircraft center of gravity location. Both decrease as the c.g. moves aft. Figure 7 shows how the stability curve changes with c.g. position.

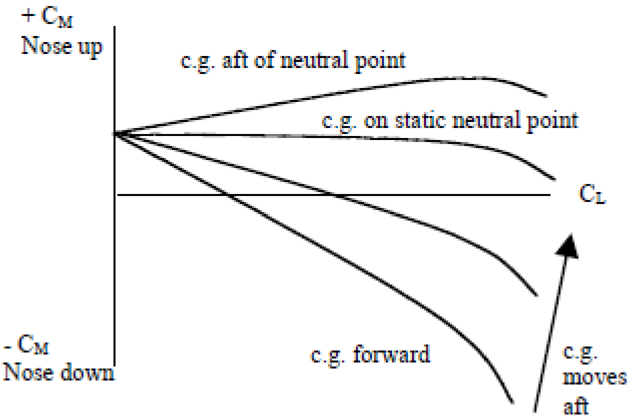


Fig. 7. Stability versus c.g.

Figure 8 shows how the curve for control force necessary to fly at airspeed other than trim varies with c.g. position.

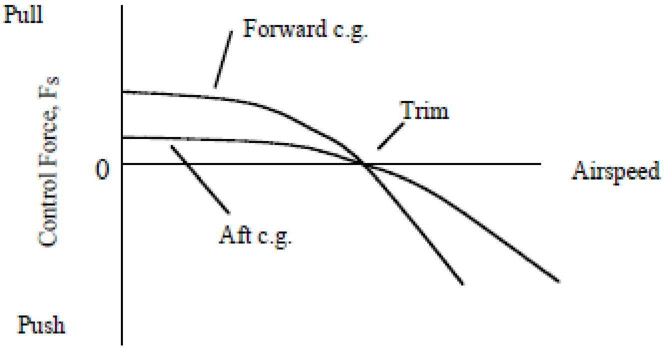


Fig. 8. Control force versus c.g.

The forces necessary are greatest at forward c.g. Note that we're switching from a stability curve, which a pilot can infer but can't experience directly, to forces and speeds that he can.

The tendency of an aircraft to return to trim speed when the controls are released is friction and c.g. dependent. As the c.g. goes aft and the force returning the stick to the trim position becomes less powerful, friction effects become more apparent.

The aircraft can appear to have nearly neutral stability within a given airspeed band when there’s appreciable friction. If you displace the stick, let the aircraft establish a new speed, and then let go, friction may prevent the elevator from returning to its original position and the aircraft from settling back to its original speed.

The speed it does settle on is called the free return speed, which for Part 23 certification must be less than or equal to ten percent of the original trim speed.

To determine free return speeds, trim your aircraft for cruise and then raise the nose, allowing speed to stabilize about 15 knots slower.

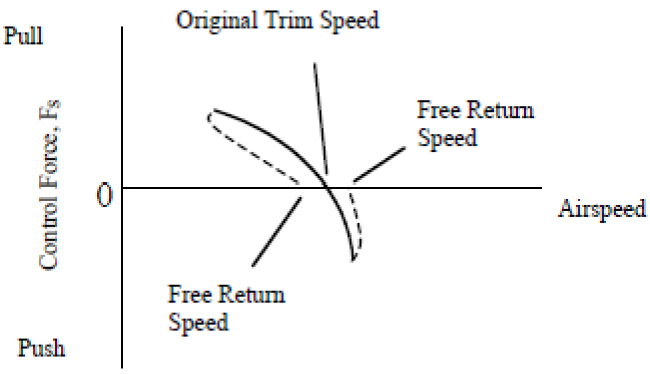


Fig. 9. Free Return Speed.

Then slowly, so as not to provoke the phugoid, release aft pressure to lower the nose back down to trim attitude and hold it there, gradually releasing aft pressure as necessary. (Don’t push, since this immediately wipes out the friction—the effect of which you’re trying to measure.) When you’ve released all aft pressure, note the speed. Repeat the exercise with a push.

First let the aircraft accelerate 15 knots, and then release forward pressure to bring the nose slowly back up to trim attitude. (Don’t pull—friction, again) Hold that attitude and note the speed at which the necessary push force disappears. The numbers show your free return trim speed band, and may explain why you’re always fussing with the trim wheel! A wide band makes an aircraft difficult to trim.

The trim speed band may become wider as the c.g. moves aft. Aft movement reduces stability, which in turn causes the slope of the control force curve to become less negative (Figure 8). Less return force is then generated to oppose the friction within the system.

Neutral Points

When the angle of attack of an aircraft changes, the net change in lift generated by the wings, stabilizer, and fuselage acts at the neutral point. The neutral point is sometimes referred to as the aerodynamic center of the aircraft as a whole, similar to the more familiar aerodynamic center of a wing. There’s no moment change about the neutral point (or about wing aerodynamic center) as angle of attack changes—only a change in lift force.

In order for an aircraft to be longitudinally stable, the center of gravity must be ahead of the neutral point. Given that condition, the top left of Figure 10 shows what happens when a gust or a pilot input increases angle of attack, α, above trim. The increased lift, acting at the neutral point some distance from the c.g., generates a stabilizing, nose-down pitching moment around the c.g. A stabilizing, nose-up pitching moment occurs if α goes down.

The aircraft on the right shows the unstable response when the c.g. lies behind the neutral point.

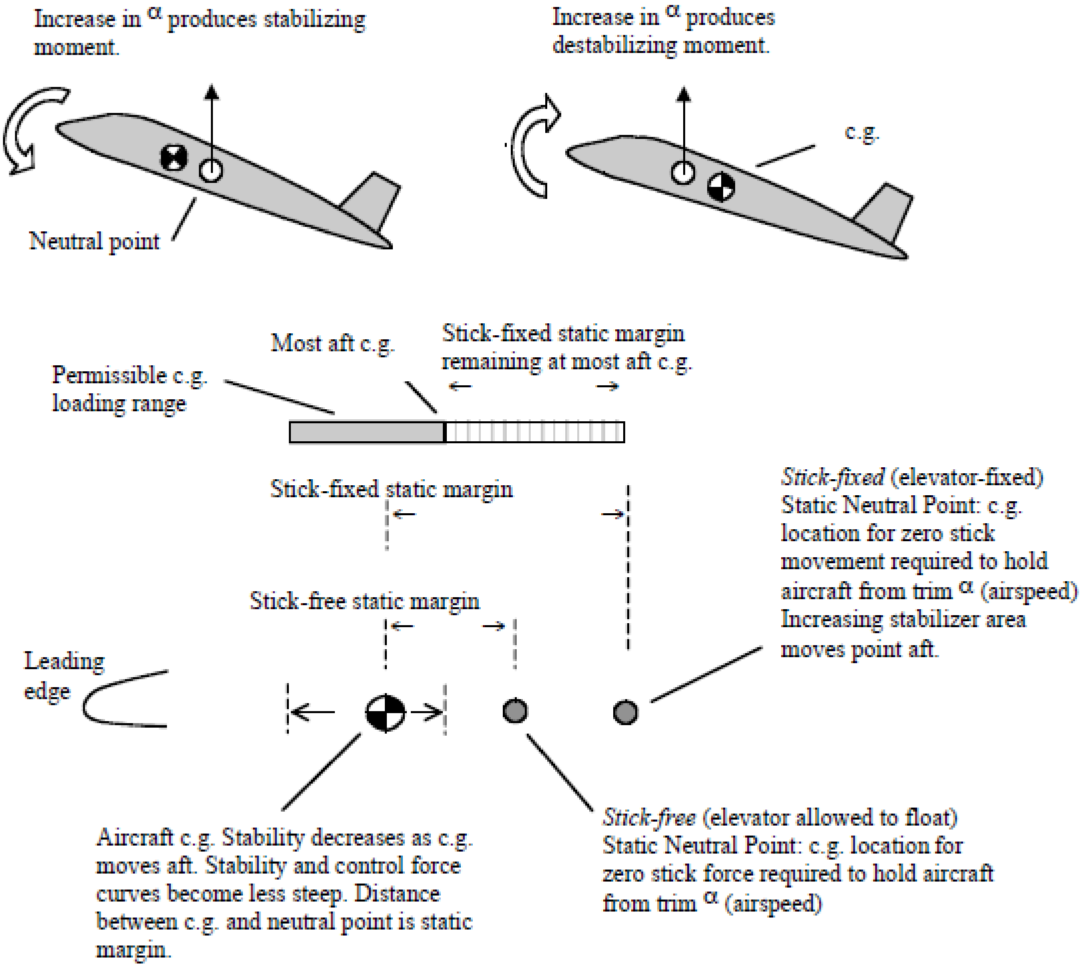


Fig. 10. Static Neutral Points.

Static stability decreases as the c.g. moves aft, toward the neutral point. The stability curve becomes increasingly flat. If you shift the c.g. all the way back to the neutral point, there'll be a change in lift whenever α changes, but no moment change. With the c.g. at the neutral point, pitching moment, C_m, becomes independent of α. The aircraft will have neutral static stability. Since the aircraft no longer generates a stabilizing moment, the pilot feels no opposing force in the stick when he moves it to fly at a new C_L.

The aircraft becomes statically unstable when the elephant finally gets loose and moves the c.g. aft of the neutral point. Once again there's a change in moment around the c.g. when α changes, but now it's destabilizing.

On a statically stable aircraft, the distance between the most permissible aft c.g. and the neutral point (both of which are expressed as percentages of the mean aerodynamic chord of the wing) is known as the static margin. The greater the static margin, the greater the stability becomes (and thus the more negative the slope of the stability curve).

Actually, as Figure 10 indicates, there're two static stability neutral points: stick-fixed (elevator and trim tab held in the prevailing trim position), and stick-free (hands off, elevator allowed to float in streamline as the angle of attack at the tail changes).

In flight-testing, stick-fixed stability determines the amount of control and elevator movement needed to change airspeed (or C_L, or α) from trim. Stick-free stability determines the required force. We'll amplify this below.

Stick-fixed Neutral Point

With a powered, irreversible control system the elevator usually doesn't float unless something broke, and so only the stick-fixed stability normally matters. (However, sometimes a programmed, artificial float is introduced to cure stability problems. Also, a control system can revert in case of hydraulic failure. The Boeing 737 reverts to a reversible system following hydraulic failure. Its predecessor, the 707, was reversible to begin with.)

At a given center of gravity position, an aircraft's static stick-fixed stability is proportional to the rate of change of elevator angle with respect to aircraft lift coefficient (aircraft lift coefficient includes the combined wing and fuselage lift effects). In other words, the more stable the aircraft is (the larger the static margin) the farther you have to haul back or push on the stick. As you bring the c.g. back, less stick movement is needed to produce an equivalent change in C_L and airspeed—and less spinning of the trim wheel is necessary to trim out the resulting forces. If the c.g. is brought back to the stick-fixed static neutral point, the change in stick position needed to sustain a change of airspeed is zero. Once you've moved the stick to attain a new angle of attack, you can put it back to where it was before.

Reportedly, the Spitfire has just about neutral stick-fixed static stability in all flight modes. The DC-3 is stable in power-off glides or at cruise power but unstable at full power or in a power approach at an aft c.g.

Stick-free Neutral Point

The stick-free static neutral point is the c.g. position at which the aircraft exhibits neutral static stability (slope of the $\Delta CM/\Delta CL$ stability curve = 0) with the elevator allowed to float. In other words, it's the position where pitching moment, CM, is independent of CL with the stick left free.

Your intuition may tell you that stick-fixed static stability is likely to be greater than the elevator-floppy situation of stick-free, because of the fixed elevator's greater efficiency in producing restoring pitching moments. The actual difference between fixed and free in an aircraft with reversible controls (with reversible controls, wiggling the control surface wiggles the stick) depends on elevator control system design, in particular the control surface hinge moments.

Aerodynamic balance used to reduce hinge moments, and thus reduce the force a pilot has to apply to deflect the elevator, also reduces floating tendency—and therefore increases the stick-free static stability margin. The stick-free neutral point usually lies ahead of the stick-fixed point. Just how far ahead depends directly on how much the elevator tends to float.

Figure 8 showed how the longitudinal stick force, FS, necessary to move an aircraft off its trim point decreases as the center of gravity moves aft. This is the logical result of the accompanying decrease in static stability. When the aircraft's c.g. lies on the stick-free neutral point, no change in force is needed to change airspeeds.

Conventional handling qualities require that the aircraft c.g. lie ahead of the stick-free static neutral point. If c.g. moves behind the neutral point, control forces reverse. A pull force becomes necessary to hold the aircraft in a dive; a push force becomes necessary in a climb.

Longitudinal Maneuvering Stability

Maneuvering Stability

Longitudinal maneuvering stability is really just static stability with an additional factor: pitch rate. An aircraft in accelerated (curved) flight—whether pulling up, pushing over, or turning—has a pitch rate. Figure 11 shows the simple case of an aircraft in a pull-up. The aircraft pitches about its c.g. The tail sweeps along behind, on its arm, l_T. The tail's motion creates a change in its relative wind and thus in tail angle of attack, α_T. The change in tail angle of attack due to pitch rate produces an opposing pitching moment, known as pitch damping.

The change in tail angle of attack, ΔαT, due to pitch rate is shown in the formula below, where q is pitch rate in radians per second (one radian equals 57.3°; and 0.1 radian/second is approximately 1 RPM). l_T is the distance between aircraft c.g. and the aerodynamic center of the tail. V_T is the velocity of the tail (taken tangentially to the aircraft's flight path). Thus the faster you pitch, and/or the farther back your tail, the greater the change in αT, but it's all inversely proportional to speed, V_T, as the formula shows.

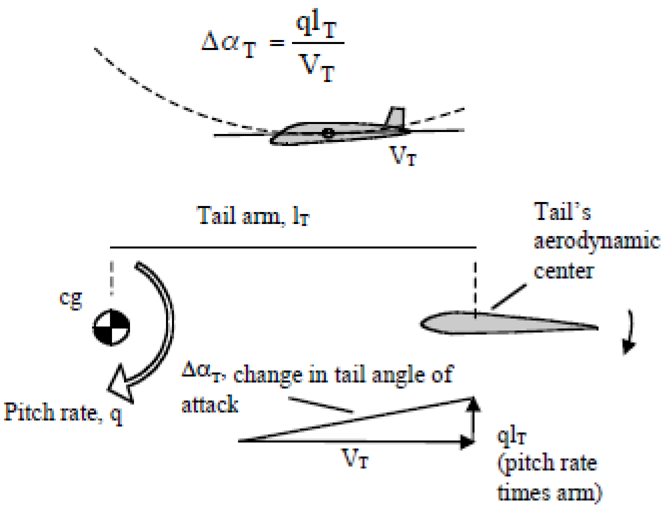


Fig. 11. Pitch Damping.

The actual tail angle of attack will also depend on the increased downwash produced by the wing as its lift coefficient rises in the pull-up, and a proper formula would take that into account.

Because of pitch damping, an aircraft is actually more stable in maneuvering flight than in flight at 1-g. Remember, we assess stability in terms of the force needed to displace the aircraft from equilibrium (trim). We assess static stability in terms of the push or pull on the stick necessary to change the coefficient of lift, CL, and to produce airspeeds different than trim, while flying at 1-g.

In maneuvering flight at more than 1-g, pitch damping increases the stick force we have to apply to displace the aircraft from equilibrium. How rapidly stick forces will increase as we increase g depends on the maneuvering characteristics for which the aircraft was designed, and its c.g. location. We can examine an aircraft's stick-fixed (elevator position-per-g) and the really more germane—since it's what the pilot feels—stick-free (stick force-per-g) maneuvering characteristics.

Figure 12 shows how the gradient, or slope, of stick force-per-g depends on the location of the aircraft c.g. Forward c.g. increases an aircraft's maneuvering stability, and therefore stick forces become heavier.

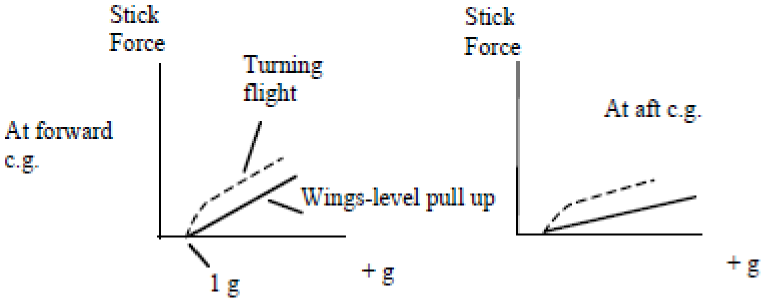


Fig. 12. Stick force per-g gradient versus c.g.

As you move the c.g. back, stick forces required to pull g go down. (The stick position-per-g curve behaves similarly. As c.g. moves aft, the deflection required to pull g goes down.)

Stick force-per-g also varies directly with wing loading (aircraft weight divided by wing area). Highly wing loaded aircraft may need the help of a powered control system to keep forces in check. Raising the wing loading has the same effect as moving the c.g. forward.

Stick force-per-g is a particularly important parameter and one of the basic handling quality differences between aircraft designed for different missions. When we maneuver an aircraft, we tend to evaluate its response in terms of the force we apply to the stick rather than the change in stick position. We know the stick has returned to the equilibrium trim position, for example, when the force disappears (at least ideally—friction and other factors can get in the way). And when we move the c.g. well aft in an aircraft—or take that first aerobatic flight—it's the reduction in stick forces we probably notice first.

Fighters and aerobatic aircraft require lower forces-per-g than do normal or transport category aircraft because their g envelopes are wider and the total stick force necessary at high g would otherwise be too great for the pilot to sustain. So a fighter operating at up to 9-g or more needs a shallower force-per-g gradient than a transport expected to operate at no greater than the 1.5-g approximately required for a 45-degree-bank level turn. The fighter's shallow force-per-g gradient would be devastating in a transport because the pilot could easily overstress the aircraft. The transport's steeper gradient would have the fighter pilot pulling with both hands while pushing on the instrument panel with his feet.

The importance of stick force-per-g in fighters became apparent during World War II. It was decided that the upper limit should be about 8 lb/g to keep the pilot from tiring in a fight, with a lower limit of 3 lb/g to prevent overstressing the aircraft and losing by default.

Overstress is the big worry; so FAR Part 23.155 specifies the minimum total control force necessary to reach an aircraft's positive limit maneuvering load factor (g limit). It's based on aircraft weight and the type of control. For wheel controls the minimum force has to be at least 1% of the aircraft's maximum weight or 20 pounds, whichever is greater, but doesn't have to exceed 50 pounds. For stick controls, minimum force for maximum g has to be at least max weight/140, or 15 pounds, whichever is greater, but doesn't have to exceed 35 pounds.

To figure out what that would mean in terms of required average minimum control-force-per-g gradient, you can take the design load limit of the airplane (6-g's for our trainers), subtract 1-g to get the maximum g-load actually applied, and then divide that into the minimum total force required by regulation. For the Air Wolf (6-g's and 2900 lbs. maximum aerobatic weight):

$$\frac{2900 \big/ 140}{5} = 4.1 \ \text{lb/g minimum allowable stick force}$$

A Cessna 172's yoke force is greater than 20lb/g. A wings-level, 1.7-g pull-up in a Boeing 777 requires 135 pounds. The Boeing is certified under FAR Part 25, which actually doesn't contain sustained maneuvering control force requirements.

The FARs doesn't specify maximum stick-force per-g, but the military does, depending on the type of aircraft.

Aircraft with shallow stick force-per-g gradients can feel dramatically sensitive if your muscle memory expects greater forces. Even experienced aerobatic pilots stepping up to higher performance aerobatic aircraft usually find themselves pulling too hard, detaching the boundary layer, and buffeting the aircraft—especially in the excitement of aerobatic competition. This is seen from the ground as an abrupt flattening in the arc of a loop, and from the cockpit as a sudden g-break. But after one becomes accustomed to those shallow gradients, the lower performance aerobatic aircraft one trained in can seem disagreeably reluctant to maneuver. The physical effort now feels out of proportion to the result.

On the other hand, pilots of early swept wing fighters had to worry about "g-limit overshoot" because of the forward shift in the center of lift as the tips began to stall. The F-86E Sabre Aircraft Operating Instructions cautioned pilots against "A basic characteristic toward longitudinal instability under conditions of high load factor, which … results in a tendency to automatically increase the rate of turn or pull-up to the point where the limit load factor may be exceeded." Fortunately, this was preceded by lots of warning buffet.

As noted, pitch damping depends on pitch rate. Pitch rate depends not just on how hard you pull, but also on the kind of maneuver you're pulling in. At a given load factor, n, (where n = lift/weight) a level turn actually requires a higher pitch rate than a wings-level pull-up.

For a level (constant altitude) turn at a given velocity, pitch rate is a function of n - 1/n, but for a wings-level pull-up it's the smaller function of n - 1. That greater pitch rate in the level turn means more pitch damping. As a result a 2-g turn, for example, requires more stick force than a 2-g pull-up. Accordingly, a high-performance turn takes more pilot muscle than a loop entry at the same load factor. See the dotted versus the solid lines in Figure 12.

Our trainers have reversible controls (wiggle an elevator by hand and the stick wiggles as well). In aircraft with reversible controls, at any given altitude and c.g., the gradient of the stick force per- g curve is independent of airspeed. Figure 13 shows how the gradient remains constant as airspeed shifts from trim. The figure also shows how the absolute stick force needed to obtain a given g will depend on the relationship between trim speed and actual airspeed. For example, when the aircraft is flying slower than trim, static stability leads to a nose-down pitching moment, which adds to the pull force a pilot has to hold to maintain a given g.

But when flying faster than trim, static stability leads to a nose-up pitching moment that decreases the pull force necessary to maintain a given g.

Because of the change in absolute stick force necessary to hold a given g at speeds slower or faster than trim, test pilots try to maintain trim speed when examining stick force-per-g in "windup turns." Otherwise the data would plot an inaccurate stick force-per-g gradient.

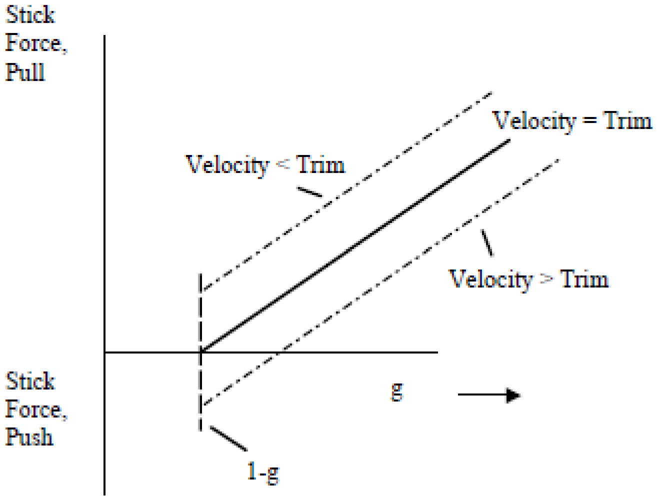


Fig. 13. Stick force- per-g gradient is constant at constant cg and altitude.

The stick force needed to pull a given g remains the same at any trim speed. Say the trim speed rises. Because the elevator's effectiveness increases with airspeed, you don't have to deflect it as much to produce a given pitch rate and load factor as you do at lower speeds. Less deflection would mean lower forces, except that control surface hinge moments—which are what the pilot feels through the control system gearing—also increase with airspeed. The decrease in required deflection is canceled out by the increase in hinge moment, and the stick force required for a given g load is the same at all trim velocities (at a constant altitude and c.g.). This holds as long as compressibility effects associated with high Mach numbers don't become a factor. Compressibility tends to produce an increase in stick force-per-g.

Damping versus Altitude

While static stability is not a function of altitude, maneuvering stability is. Stick force-per-g goes down as you go up. That's because damping decreases along with the decrease in air density as you climb.

At least that's the short explanation. Actually, in responding to a given control input an airplane doesn't care about altitude, it cares about airspeed. Compressibility effects aside, for a given input it will generate the same pitching (or rolling or yawing) moment at a given EAS (equivalent airspeed, meaning calibrated airspeed corrected for compressibility) regardless of whether it's flying down low or up high. But the damping this moment has to overcome is a function of altitude, because damping is a function of TAS (true airspeed, or equivalent airspeed corrected for density altitude), as Figure 14 explains. TAS goes up as altitude increases.

The figure shows that for a given pitch rate, q, the velocity component generated by the movement of the tail, ql_T, is the same regardless of altitude. But since true airspeed is higher at altitude, the vectors add up to less change in tail angle of attack, and so less damping.

This is why an airplane will feel more responsive and less stable at altitude, or perhaps even lower down on a hot, high-density-altitude day.

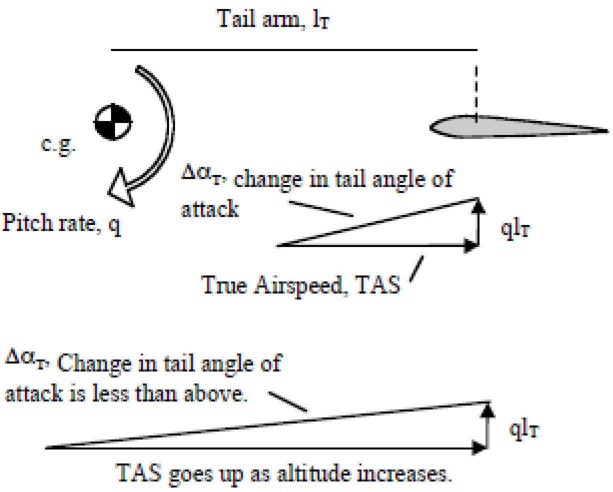


Fig. 14. Damping and TAS.

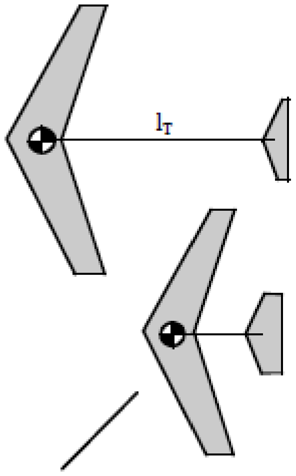
The reduction in damping also applies to an aircraft's directional and lateral stability. Stability augmentation systems, like yaw dampers, earn their keep up high.

Tail Volume

Stability depends on the restoring moment supplied by the horizontal tail being greater than the destabilizing moments caused by the other parts of the aircraft. One factor is the tail-volume coefficient, $\bar{V}$. This is the product of the distance between the aircraft c.g. and the tail's aerodynamic center, l_T, times the tail area, S_T. The result is then divided by the mean aerodynamic chord of the wing, $\bar{c}$, times the wing area, S.

$$\overline{V} = \frac{l_T S_T}{\overline{c} S}$$

In other words, the tail volume coefficient relates the area of the tail and its distance from the c.g. to the chord and area of the wing. It suggests how effective the tail is going to be at producing pitching moments.



Same tail volume coefficient as above, but shorter l_T.
Less pitch damping makes the aircraft more maneuverable.

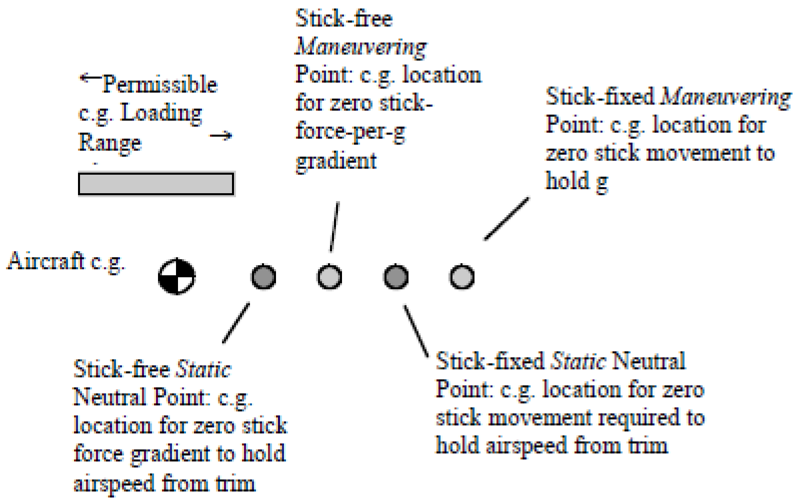
Fig. 15. Tail Volume Coefficient.

You can achieve a given tail volume for a wing of a given size either by having a small tail on a long fuselage, or a large tail on a short fuselage (Figure 15).

Since pitch damping is a function of the square of the tail's lever arm, l_T^2, the farther back your tail is the greater the opposing aerodynamic damping generated when you start pitching it around to maneuver. The design criterion for rapid maneuvering is a big tail on a short fuselage—a hallmark of modern fighter design. Transports have proportionately smaller tails on longer fuselages.

Neutral Points Again

Figure 16 adds the stick-fixed maneuver neutral point and the stick-free maneuver neutral point to the stick-fixed and stick-free static neutral points discussed in the ground school briefing "Longitudinal Static Stability". The aft shift of the corresponding maneuver points reflects the stabilizing effect of pitch damping. Because damping goes down with altitude, the maneuver points actually sneak forward as you climb.



Typically for inherent stability and good handling qualities for an aircraft with reversible controls, maximum permissible aft c.g. must be ahead of all static and maneuvering neutral points, and forward of the point for minimum allowable stick-force-per-g. Maximum forward c.g. is determined by control authority need to raise the nose to C_{Lmax}, or by the maximum allowable stick-force-per-g.

Fig. 16. Stick-fixed and free Neutral Points.

The stick-free maneuver point is the c.g. position at which the gradient of stick force-per-g becomes zero. The more rearward stick-fixed maneuver point is the c.g. position at which stick movement-per-g becomes zero.

If we had a weight on rails and could move the c.g. rearward during flight, the first thing we'd notice is a reduction in control force necessary to change α and thus airspeed from trim (static stability), accompanied by a reduction in stick force needed to pull g (maneuvering stability). Short of shifting the c.g., a knowledgeable instructor can simulate this for a student by manipulating the trim. As tail volume increases, the neutral points move aft. This in turn increases the aft c.g. loading range.

Longitudinal Dynamic Stability

Introduction to Stability

Stability is the general term for the tendency of an object to return to equilibrium if displaced.

Static stability is an object's initial tendency upon displacement. An object with an initial tendency to return to equilibrium is said to have positive static stability. For those blessed with a conventional pilot's education, the concept of stability normally evokes the textbook image of a marble rolling around in something like a teacup, as shown in Figure 17.

An airplane can't be trimmed unless it has longitudinal (around the y axis) static stability—in other words, unless pitching forces tending to equilibrium are present. But the greater an aircraft's static stability (thus the greater the forces tending to equilibrium) the more resistant the aircraft is to the displacement required in maneuvering.

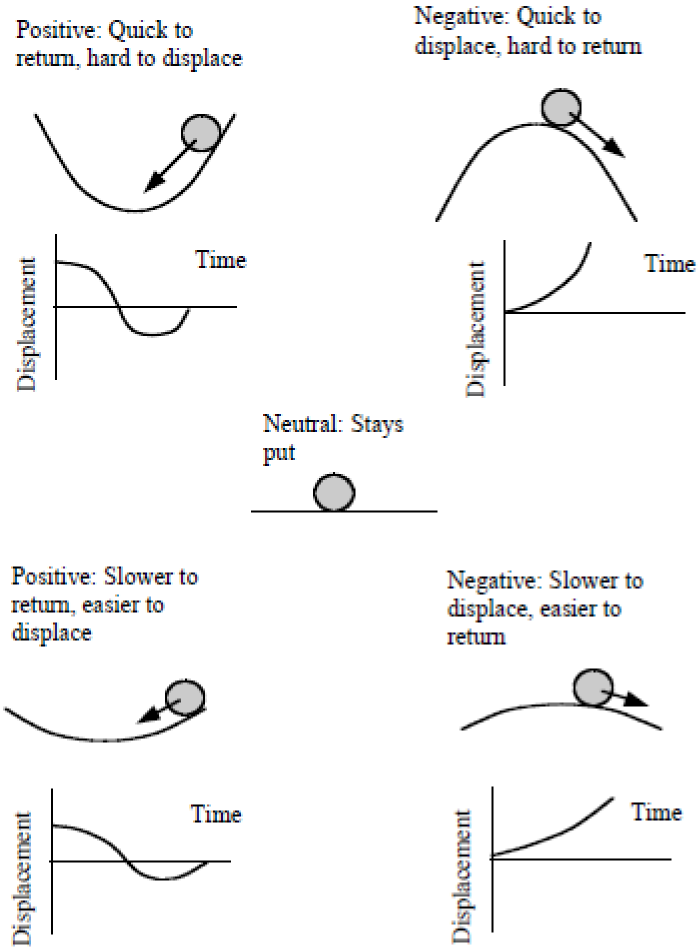


Fig. 17. Static Stability.

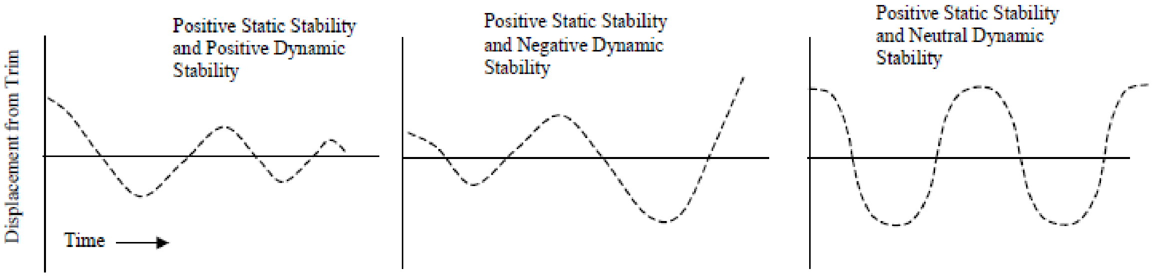


Fig. 18. Dynamic Stability.

For a given aircraft, the most important factor in determining longitudinal static stability is c.g. position. Moving the c.g. aft reduces static stability.

Dynamic stability, our real subject here, refers to the time history that transpires following displacement from equilibrium, as shown in Figures 17 and 18.

Aircraft can either have inherent aerodynamic stability (the typical case), or de-facto stability, in which stability requirements are met with the aid of a control system augmented with sensors and feedback. For example, in order to achieve maximum maneuverability, the F-18 lacks inherent stability, and can't be flown without some operational brainpower on board in addition to the pilot. The Boeing 777 has relaxed inherent longitudinal static stability, which produces efficiencies in cruise from a more rearward c.g. and a physically lighter tail structure than otherwise possible.

Boeing transport aircraft have conventional downward lifting tails that, like all such tails, in effect add weight to the aircraft by virtue of the "down-lift" they generate (and also drag, the by-product of that lift). The main wing has to produce additional lift in compensation, and consequently produces more drag itself, which costs money at the gas truck. Moving the c.g. aft reduces the necessary down-force. The 777's digital flight computers make up for the resulting longitudinal stability deficit—but the aircraft still has to have sufficient inherent stability to be flown safely and landed should the digital augmentation go bust. The monster Airbus A380 employs an aft center of gravity for the same reason. It can pump fuel aft to shift the c.g.

Dynamic Stability: Short Period and Phugoid

Figure 19 illustrates positive longitudinal dynamic stability: a series of damped oscillations of constant period, or frequency, and diminishing amplitude that bring the aircraft back to the trimmed condition after a displacement.

Period is time per cycle. Frequency, which is inversely proportional to period, is cycles per unit of time. Amplitude is the difference between the crest or the trough and the original equilibrium condition.

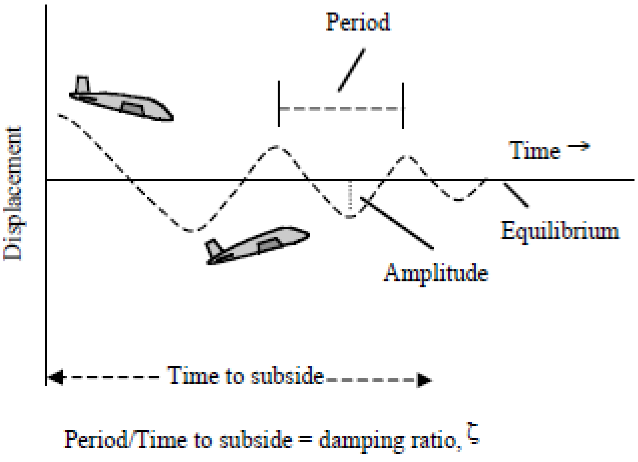


Fig. 19. Positive Dynamic Stability.

Damping is the force that decreases the amplitude of the oscillation with each cycle. The damping ratio, ζ, is the time for one cycle divided by the total time it takes for the oscillation to subside. The higher the damping ratio, the more quickly the motion disappears. Damping defines much about the character of an aircraft. If damping is too high, an aircraft may seem sluggish in response to control inputs. If damping is too low, turbulence or control inputs can excite the aircraft too readily; its behavior appears skittish.

There are two modes of pitch oscillation: the heavily damped short period mode (damping ratio about 0.3 or greater), followed by the lightly damped, and more familiar, long period, phugoid mode. When you maneuver an airplane in pitch by moving the stick forward or back, you initially excite—and essentially just ride through—the short period mode. If you were then to let go or to return the stick back to the trim position, the aircraft would enter the phugoid mode. Instead, you normally hold the pressures necessary to prevent a phugoid from occurring.

Short Period

The short period mode is excited by a change in angle of attack. The change could be caused by a sudden gust or by a longitudinal displacement of the stick. Figure 20 shows the variation in angle of attack, α, over time. The aircraft quickly overshoots and recovers its original angle of attack, or its new angle of attack in the case of an intentional pilot input and a new stick position. The motion of the tail causes most of the damping, although other parts of the aircraft can contribute to damping (or to oscillation). There's negligible change in altitude or airspeed by the time the mode subsides. During the short period oscillation the aircraft pitches around its c.g.

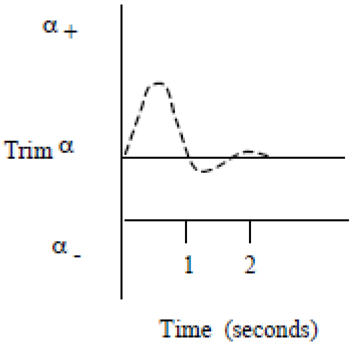


Fig. 20. Short Period

Positive damping of the short period is important because catastrophic flight loads could quickly build from a divergent oscillation—suddenly the airplane has oscillated into parts. The short period mode is also an area in which pilot induced-oscillations, PIO, can occur, because the typical lag time in pilot response is about the same as the mode's period (approximately 1-2 seconds). As a result, by the time a pilot responds to an oscillation his control input is out of phase, and he ends up reinforcing rather than counteracting the motion he's trying to correct.

At some point during our flights, we can perform a frequency sweep with the stick to try to isolate the aircraft's short period natural frequency, ωn. (As a child you pumped a swing in rhythm with its natural frequency to make it go higher and terrify your mother.) We'll do this by moving the stick back and forth over a constant deflection range of perhaps three or four inches, but faster and faster until we find the input frequency that places us 90 degrees out of phase—meaning that the stick is either forward or back when the nose is on the horizon (although it can be hard to tell).

We're then at the undamped short period natural frequency—undamped because we're driving it with the stick. Then we'll abruptly return the stick to neutral when the aircraft is at its trim attitude, and observe the damping of the short period oscillation. It subsides very quickly, as in Figure 20.

The frequency sweep is not occupant friendly, but it's a good way to assess an aircraft's pitch acceleration, or "quickness." The high pitch acceleration—the ability to quickly change angle of attack—is one of the first things you'll notice when transitioning to high-performance aerobatic aircraft. You can think of an aircraft's natural frequency in terms of its ability to "follow orders"—how rapidly you can tell it to do one thing, then tell it the opposite, and still have it respond. The higher the natural frequency, the more response cycles you can coax from it per unit of time. As we do our sweep, you'll notice that past a certain point you can't coax any more. Then the faster you move the controls back and forth the less the aircraft responds. It's as if the aircraft figures that you can't make up your mind, and that you need to be ignored.

An aircraft with a low natural frequency may seem initially unresponsive to control input. A pilot may then over control, using a large initial input to get things going, only to find that the aircraft's final response is excessive. If the natural frequency is too high, the aircraft will feel too sensitive in maneuvering and too responsive to turbulence.

Aircraft with low short period damping ratios tend to be easily excited by control inputs and turbulence, and the resulting oscillations take longer to disappear. Aircraft with high short period damping can be slow to respond—they're sluggish, and the control forces seem high. (We'll also look at our trainer's quickness in roll.

The notion of a natural roll frequency doesn't really apply, because an aircraft isn't supposed to oscillate in roll. Oscillatory response is characteristic of "second-order" systems. First order systems, like a rolling aircraft, are exponential and non-oscillatory. We'll do some "roll sweeps," anyway. You'll discover a similar fall-off in response.)

Long Period—Phugoid

The lightly damped, long period, or phugoid, oscillation can take minutes to play out. But it doesn't get to very often. Unlike the short mode, the phugoid period is long enough that the pilot can intervene easily and return the aircraft to equilibrium. We typically demonstrate the phugoid by pitching the nose up (thus exciting the short period mode) and allowing the aircraft to decelerate and stabilize some 15-20 knots below trim. Then we positively return the stick to its original trimmed position. The positive return overcomes any friction in the elevator system, and this keeps us from imposing an overall descent or a climb onto the phugoid.

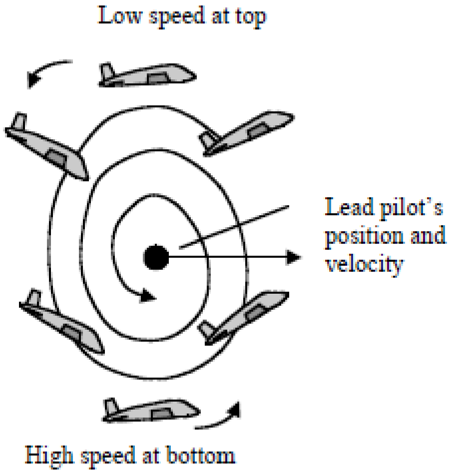


Fig. 21. Phugoid Wingman

Usually it doesn't matter if you then hold the stick or let it go, since the difference between stick-fixed and stick-free is minor in the long period mode. But for consistency in response we want to keep the wings level. By using rudder for that job, we can avoid inadvertent pitch inputs. (On our flights we'll often enter a phugoid more theatrically, perhaps as the recovery from a spiral dive.)

From the nose-high attitude, the nose will begin to drop through the horizon into a descent, then pitch up and climb back up as speed increases. It then repeats the cycle, overshooting its original, trimmed airspeed/altitude point by less and less each time. During the phugoid the aircraft maintains essentially a constant angle of attack, α, while cyclically trading altitude and airspeed (potential and kinetic energy) until it again regains equilibrium as in (Figure 19). The pitch rate and the variation in maximum pitch attitude will diminish with each oscillation. Pitch attitude at the very top and bottom will be approximately the same as the original pitch attitude at trim.

Minimum airspeed will occur at the point of maximum altitude, and maximum airspeed will occur at the point of minimum altitude. The phugoid oscillation is typically damped and convergent, but it can be neutral, or even divergent, and the aircraft will still be flyable, because of the ease with which the pilot can bring the long period under control (you're controlling the phugoid whenever you hold altitude). But poor damping does increase the workload and complexity of the scan for instrument pilots when flying by hand, because the effort needed to hold altitude increases. Poor damping also makes it harder to trim an aircraft.

The undulating lines back in Figure 19 suggest how the phugoid would appear to a stationary observer. Figure 21 shows the same from the point of view of another pilot flying level in formation, watching a "phugoidal" ("phugoiding?") wingman. The aircraft appears to rise and fall as airspeed changes produce lift changes. Excess airspeed at the bottom produces lift greater than weight and a resulting upward force. Diminished airspeed at the top produces lift less than weight and a resulting downward force. Remember, α stays the same.

As a result of the airspeed changes an aircraft in the phugoid would also appear to move back and forth, falling behind at the top of the cycle and scooting forward at the bottom, but less and less each time as the motion damps out.

Drag effects, rather than tail movement, damp the phugoid. Raising parasite drag increases damping. With both the short period and the phugoid mode, an aft shift in c.g., close to the neutral point, will begin to produce an increase in period and a decrease in damping (for neutral point, see ground school "Longitudinal Static Stability").

Propellers add a damping factor absent with jets. If brake horsepower is constant, propeller thrust increases as airspeed decreases, and vice versa. This adds a forward force at the low-speed top of the phugoid and a restraining force at the highspeed bottom. This changing thrust/airspeed relationship helps reduce the speed variation from trim and thus helps damp the motion.

The phugoid is sensitive to coefficient of lift, C_L. At slow speeds, thus at high C_L, both the period and the damping decrease. At high speeds, thus at low C_L, both period and damping increase.